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AGRICULTURAL MEMORANDUM 969

Calculation of air trajectories over the sea

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INTRODUCTION

The Agricultural Section of the Meteorological Office require a simple model for predicting the direction and speed of movement of airborne virus particles (eg Foot-and-mouth disease virus), pollen spores and insects over both land and sea. This information is needed to improve advice, both in the forecast of disease spread and in the analysis of past outbreaks, given by the Meteorological Office to, amongst others, the Ministry of Agriculture.

The Meteorological Office Boundary Layer Research Branch were asked to calculate the movement of a plume in the surface boundary layer over both land and sea in terms of a very basic but standard meteorological observation. Two programs have been written for a Texas Instruments TI59 programmable calculator; the first of these is for movement overland (Agricultural Memorandum 968) and the second over the sea.

This memorandum describes the formulae used in the second of these programs and presents the solution of these formulae for a range of input conditions. Finally the results are expressed in the terms of three general equations.

FORMULAE USED IN ADVECTION PROGRAM

Assumptions: (1) the plume has had time to disperse throughout the mixing layer (height h). Consequently the vertical dispersion coefficient of the plume (σ_z) has been taken as $0.6h$.

(2) the stability of the air over the sea rarely deviates markedly from neutral. (Augstein, E, ECMWF seminars on "The treatment of the boundary layer in numerical weather prediction", Reading 6-9-76, pp 138-182).

FH2B

Input Data:

U_{10} = 10 metre wind speed ($M\ sec^{-1}$)

θ_{10} = 10 metre wind direction (^{o})

T_S = surface sea temperature (^{o}C)

T_{10} = 10 metre air temperature (^{o}C)

Sensible Heat flux (H)

$$= \rho c_p C_H U_{10} (T_s - T_{10}) \quad \ldots\ldots\ldots (1)$$

where ρ = density of the air

c_p = specific heat of air at constant pressure

C_H = bulk transfer coefficient for heat $\simeq 10^{-3}$

Friction velocity (u_*)

$$= \sqrt{C_D \cdot U_{10}^2} \quad \ldots\ldots\ldots (2)$$

where C_D = drag coefficient = $1.25\ 10^{-3}$

Roughness length (z_o)

$$= 10\, e^{-k/\sqrt{C_D}} \quad \ldots\ldots\ldots (3)$$

$$= 1.22 \cdot 10^{-4} \text{ metres}$$

where k = Kármán constant = 0.4

Monin-Obukhov length scale (L)

$$= -\frac{94700\, u_*^3}{H} \quad \ldots\ldots\ldots (4)$$

Monin-Kazanski stability parameter (μ_*) $= \dfrac{-0.00038\, H}{u_*^2}$ $\ldots\ldots\ldots (5)$

Using Rossby similarity theory the geostrophic wind (G) can be found

$$G = \frac{u_*}{k}\sqrt{B^2(\mu_*) + \left(\ln \frac{u_*}{f z_o} - A(\mu_*)\right)^2} \quad \ldots\ldots\ldots (6)$$

where $B(\mu_*) = 5.14 + 14.2\mu_* + 11.7\mu_*^2 - 3.3\mu_*^3$ $\ldots\ldots\ldots (7)$

$A(\mu_*) = 1.01 - 10.5\mu_* - 9.9\mu_*^2 + 0.81\mu_*^3$ $\ldots\ldots\ldots (8)$

f = Coriolis parameter

The turning of the wind (α)

$$= \tan^{-1}\left[\frac{B(\mu_*)}{\left[\left(\frac{kG}{u_*}\right)^2 - B(\mu_*)\right]^{1/2}}\right] \quad \ldots\ldots\ldots (9)$$

The method used to determine the depth of the mixing layer (h) and the Pasquill stability parameter (P) depends upon the heat flux (H)

If $H < 0$ go to (a) then (c)

If $H \geq 0$ go to (b) then (c)

(a) Stable conditions

$$\frac{1}{h^3} = \frac{1}{z_N^3} + \frac{1}{z_r^3} \quad \ldots\ldots\ldots (10)$$

where z_N is an estimate of the mixing depth in neutral conditions $= \frac{u_*}{4f}$ ..... (11)

z_r is an estimate of the mixing depth in stable conditions $= \frac{21500\, u_*^2}{\sqrt{|H|}}$ ..... (12)

$$P = 3.6 + 8.0\mu_* - 8.5\mu_*^2 + 2.9\mu_*^3 \quad \ldots\ldots\ldots (13)$$

(b) Neutral or unstable conditions.

Over the sea the depth of the mixing layer is expected to be rather close to that given by eqn (11) for neutral stability. However some correction may be made according to eqn (14), an empirical interpolation formula. Over land the mixing layer evolves in response to the diurnal variation in surface heat flux. Strictly any determination of the depth should reflect this time-evolution. However, eqn (15) gives a useful approximation to the depth in terms of the current surface heat flux. Recognising its limitations we will apply this formula (eqn (15)) to the situation over the sea.

$$h^2 = z_N^2 + z_u^2 \quad \ldots\ldots\ldots (14)$$

where $z_N = \frac{u_*}{4f}$ ..... (11)

z_u = an estimate for the mixing depth in unstable conditions

$= 0.0000163\, H^3 + 0.00125\, H^2 + 2.2953\, H$ ..... (15)

P (based on experimental data from Cardington) =

$$\frac{3.6}{1 + 0.53/\mu_* + 4.9/\mu_*^2 - 2.0\mu_*^3} \quad \ldots\ldots\ldots (16)$$

(c) The appropriate advection wind (U) is then found by interpolation

$$U = G - a(G - U_{10}) \quad \ldots\ldots\ldots (17)$$

$$\text{where } a = k'x^2 + lx \quad \ldots\ldots\ldots (18)$$

$$X \equiv \ln \frac{h}{\sigma_z} = \ln \frac{1}{0.6} \quad \ldots\ldots\ldots (19)$$

$$k' = \frac{x_0 - b x_1}{x_0 x_1 (x_1 - x_0)} \quad \ldots\ldots\ldots (20)$$

$$x_0 = \ln \frac{h}{z_0} \quad \ldots\ldots\ldots (21)$$

$$x_1 = \ln \frac{h}{10} \quad \ldots\ldots\ldots (22)$$

$$b = \frac{G}{G - U_{10}} \quad \ldots\ldots\ldots (23)$$

$$l = \frac{1}{x_1} - k' x_1 \quad \ldots\ldots\ldots (24)$$

The distance (in km) travelled in 1 hour (d) = $3.6\,U$ ..... (25)

The resultant direction of plume movement (θ)

$$= \theta_{10}\left(\frac{h - \sigma_z}{h - 10}\right) + \left(\theta_{10} + \alpha\right)\left(1 - \frac{h - \sigma_z}{h - 10}\right) \quad \ldots\ldots (26)$$

SOLUTION OF FORMULAE

Values for the deviation in the direction of travel of the plume from that of the surface wind direction, $(\theta - \theta_{10})$, the associated mixing depth (h) and the distance of travel in 1 hour (d) were calculated using the equations described previously. Figure 1-3 present the results as a function of the ten metre windspeed (u_{10}) and the difference between the air and the sea temperatures $(T_{10} - T_S)$.

SIMPLIFICATION OF FIGURES 1-3

All of the results presented in Figures 1-3 can be expressed in terms of three equations:

$$\theta - \theta_{10} = \frac{21.5}{1 + e^{\left(0.03 + \left[-0.475 + 0.14 \ln u_{10}\right]\left[T_{10} - T_S\right]\right)}} \quad \ldots\ldots\ldots (27)$$

This equation gives values of $\theta - \theta_{10}$ within 0.4° of the original values over the majority of the ranges. The maximum error (1.6°) occurs at $T_a - T_s = -10^{\circ}C$ with $u_{10} = 20$ m sec^{-1}.

$$h = 28.2\, u_{10}^{1.35}\, (T_{10} - T_S)^{(-0.5 + 0.006\, u_{10})} \quad \ldots\ldots\ldots (28)$$

This equation gives values for h to better than 5% of the original values over the majority of the range. The maximum error (6.3%) occurs at $T_{10} - T_S = 20^{\circ}C$ with $u_{10} = 20$ m sec^{-1}.

$$d = (-2.82 + 5.54\, u_{10})\, e^{(0.10\, e^{-0.11})(T_{10} - T_S)} \quad \ldots\ldots\ldots (29)$$

This equation gives values for d to around 2-3% of the original values over the normal operational ranges and to better than 6% over a much wider range.

SUMMARY

A method by which the movement of a plume in the surface boundary layer over the sea can be described has been presented. Equations (1) to (26) have been programmed on a Texas Instrument TI59 calculator. These equations were then solved for a wide range of meteorological conditions and the results expressed in terms of three further equations ((27) to (29)). For practical purposes the use of either Figures 1 to 3 or the equations (27) to (29) will provide a reasonable estimate for:-

(1) the deviation of the plume from that of the surface wind

(2) the distance travelled in 1 hour by the plume

(3) the theoretical height of the top of the mixing layer at the time of observation.

Any comments or suggestions on this paper should be referred to the authors.

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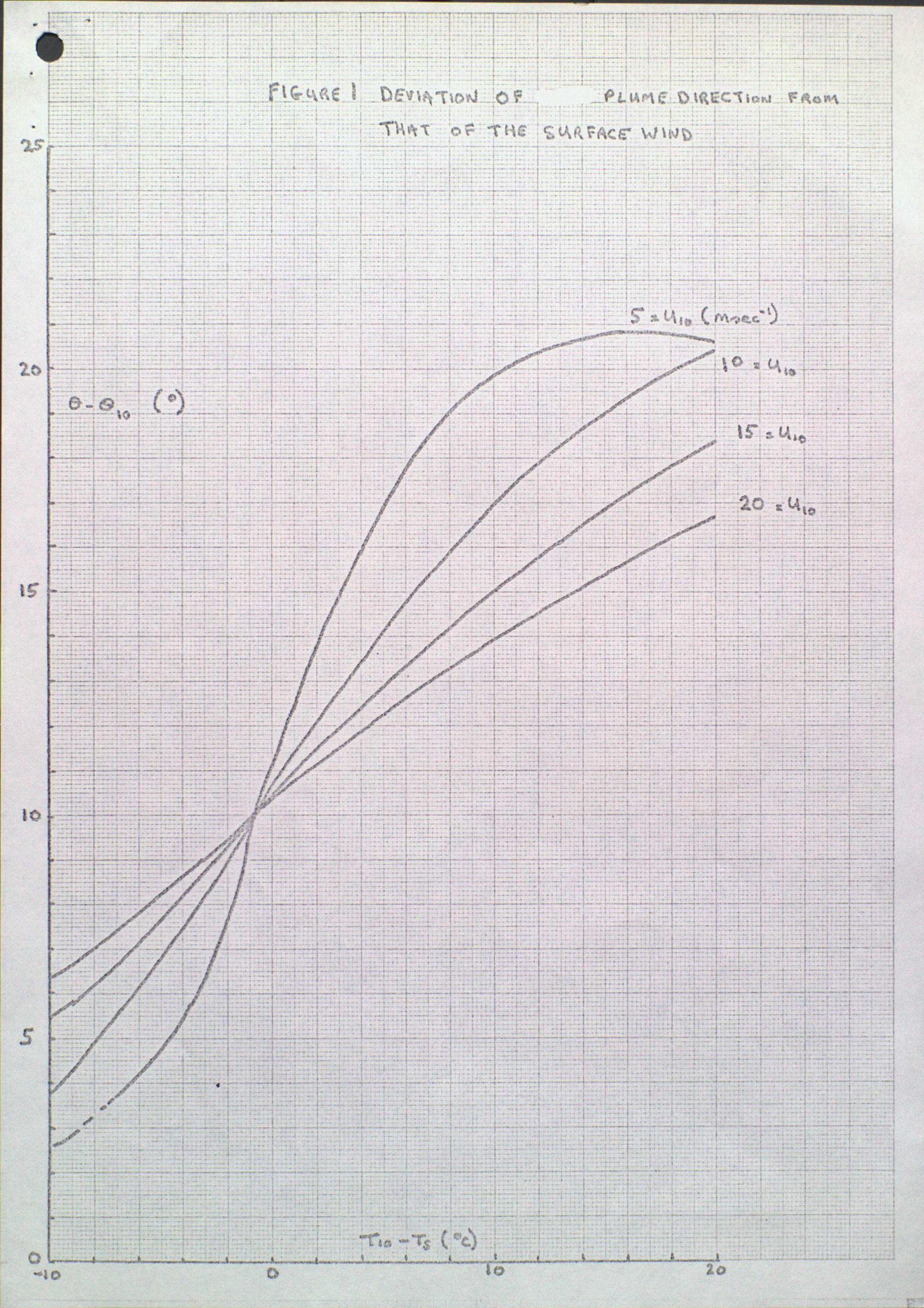
FIGURE 1 DEVIATION OF PLUME DIRECTION FROM THAT OF THE SURFACE WIND



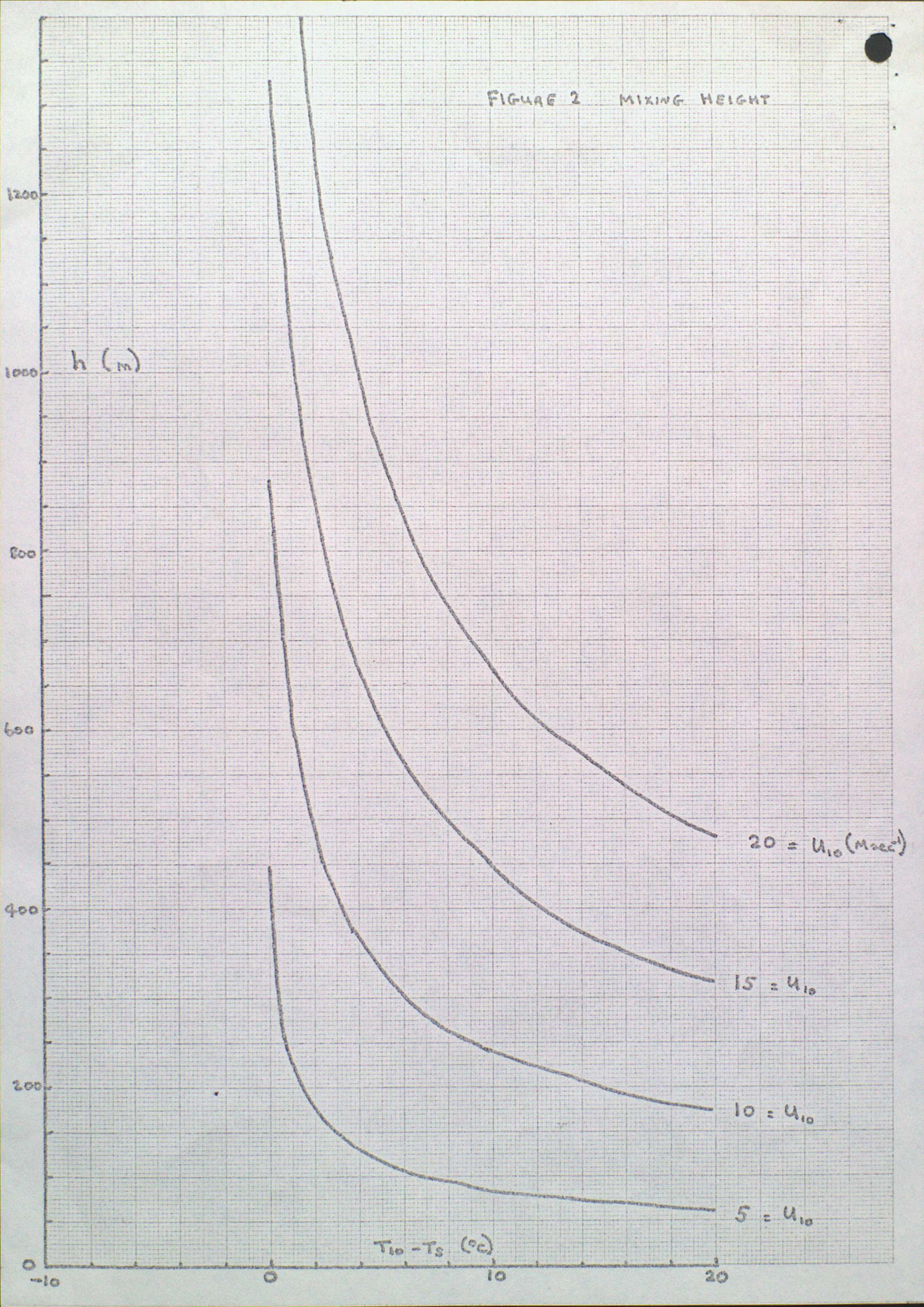


FIGURE 2 MIXING HEIGHT
h (m)
1200
1000
800
600
400
200
0
-10
0
10
20
$T_{10} - T_S$ (°C)
20 = U_{10} (M sec^{-1})
15 = U_{10}
10 = U_{10}
5 = U_{10}

FIGURE 3 DISTANCE PLUME TRAVELS IN 1 HOUR